Eur. Phys. J. C 23, 749–755 (2002)
Digital Object Identifier (DOI) 10.1007/s100520100882

THE EUROPEAN PHYSICAL JOURNAL C

Supersymmetric penguin contributions to the decay $b \to s\gamma$ with non-universal squarks masses

M.B. Causse[a], J. Orloff[b]

Laboratoire de Physique Corpusculaire, Université Blaise Pascal, 63177 Aubière Cedex, France

Received: 20 April 2001 / Revised version: 14 December 2001 /
Published online: 5 April 2002 –

Abstract. We give explicit expressions for the amplitudes associated with the supersymmetric (SUSY) contributions to the process $b \to s\gamma$ in the context of SUSY extensions of the standard model (SM) with non-universal soft SUSY breaking terms. From experimental data we deduce limits on the squark mass insertions obtained from different contributions (gluinos, neutralinos and charginos).

1 Introduction

The rare B decays represent a good test for new physics beyond the SM, since they are not affected appreciably by uncertainties due to long distance effects. Here, in the context of spontaneously broken minimal $N = 1$ supergravity [1], we study penguin diagrams with gluinos, neutralinos and charginos, which are responsible for $\Delta S = 1$ radiative mesonic decays. In particular, we study the $b \to s\gamma$ decay [2] that gives the most stringent lower bounds on the average squark mass. We know that in generic MSUGRA models [3], the soft universal breaking terms lead to a high degeneracy in the sfermionic sector. Flavor changing neutral current (FCNC) tests play an important role in constraining the SUSY mass spectrum.

We thus consider the SUSY extensions of the SM with non-universal soft breaking terms [5]. We shall use the mass insertion method by which it is possible to obtain a set of upper bounds on the off-diagonal terms (Δ) in the sfermion mass matrices (the mass terms relating sfermion of the same electric charge but different flavor). Obviously the mass insertion method offers the major advantage that one does not need the full diagonalization of the sfermion mass matrices. Then only a small number of effective parameters (δ) summarize the effects. We have applied this method to the gluinos, neutralinos and charginos contributions to the decay $b \to s\gamma$, the charginos' contribution being original. From experimental limits, we have then derived upper bounds on the off-diagonal terms in the sfermion mass matrices (for squarks down and up).

This paper is organized as follows. In Sect. 2 we present generalities on the minimal SUSY models and the mass insertion method. In Sect. 3, we give the explicit expressions of the amplitudes associated with the gluinos, neutralinos and charginos contributions to the decay $b \to s\gamma$. Finally in Sect. 4, we find explicit expressions for the branching ratio $\mathrm{BR}(b \to s\gamma)$ and the upper bounds obtained for the off-diagonal terms (Δ) in the squark mass matrices. In the appendix, we recall the analytic expressions for the Feynman integrals which arise in the evaluation of these amplitudes.

2 Generalities on the minimal SUSY models and the mass insertion method

The minimal supersymmetric standard model (MSSM), obtained by supersymmetrizing the SM field contents and allowing for all possible soft SUSY-breaking terms, contains a huge number of free parameters. In this note, we concentrate on a specific set of models in which these soft breaking terms are close to the MSUGRA universality.

By minimal supergravity models [3] (MSUGRA) below we mean the low energy limit of spontaneously broken $N = 1$ supergravity theories which supersymmetrize the SM and present the following two features:

(1) R-parity is implemented so that no baryon and/or lepton number violating terms appear in the superpotential;
(2) the Kähler metric is flat, i.e. all the scalar kinetic terms are canonical.

These features bring about new sources of FCNC (flavor changing neutral current) effects. The experimental limits on B meson physics and in particular $b \to s\gamma$, constitute interesting FCNC tests for these MSS models, typically requiring squark masses of the same electric charge to be relatively degenerate, i.e. their mass difference must be smaller than their average value. Briefly, we review the major ingredients which give rise to this new source of FCNC.

[a] e-mail: mbc@wanadoo.fr
[b] e-mail: orloff@in2p3.fr

The low energy Lagrangian consists of

(1) the superpotential of the $N = 1$ globally supersymmetric SM:

$$W = h_u Q H_1 u^c + h_d Q H_2 d^c + h_{\mathrm{L}} L H_2 e^c + \mu H_2 H_1; \quad (1)$$

(2) the scalar part of SUSY soft breaking terms for the minimal $N = 1$ supergravity theories:

$$\begin{aligned} L_{\mathrm{soft}}^{\mathrm{scal}} &= m^2 \sum_{i=\mathrm{scalar}} |\varphi_i|^2 \\ &+ \Big[Am \left(h_u \widetilde{Q} H_1 \widetilde{u^c} + h_u \widetilde{Q} H_2 \widetilde{d^c} + h_{\mathrm{L}} \widetilde{L} H_2 \widetilde{e^c} \right) \\ &+ Bm\mu H_2 H_1 + \mathrm{h.c.} \Big], \end{aligned} \quad (2)$$

where A and B are two dimensionless free parameters of the trilinear and bilinear scalar contributions; m denotes the scale of the low energy SUSY breaking.

From (1) and (2), we obtain the 6×6 squark down mass matrix ($Q = -1/3$)

$$M^2_{\widetilde{dd^*}} = \begin{bmatrix} m^2_{\widetilde{d_{\mathrm{L}} d^*_{\mathrm{L}}}} & m^2_{\widetilde{d_{\mathrm{L}} d^c_{\mathrm{L}}}} \\ m^2_{\widetilde{d^*_{\mathrm{L}} d^{c*}_{\mathrm{L}}}} & m^2_{\widetilde{d^c_{\mathrm{L}} d^{c*}_{\mathrm{L}}}} \end{bmatrix}, \quad (3)$$

where

$$m^2_{\widetilde{d_{\mathrm{L}} d^*_{\mathrm{L}}}} = m^2_{\widetilde{d^c_{\mathrm{L}} d^{c*}_{\mathrm{L}}}} = m_d m_d^+ + m^2 \times 1 \quad (4)$$

and

$$m^2_{\widetilde{d_{\mathrm{L}} d^c_{\mathrm{L}}}} = Amm_d + \mu m_d \langle H_1 \rangle / \langle H_2 \rangle, \quad (5)$$

with $m_d = 3 \times 3$ D quarks mass matrix and $e_D = -1/3$ is the electric charge.

At this stage, it is clear that the d_{L}–$\widetilde{d^+_{\mathrm{L}}}$–$\widetilde{g}$ coupling cannot lead to flavor changes (FC). Indeed, if we diagonalize $m_d m_d^+$, we diagonalize at the same time $m^2_{\widetilde{d_{\mathrm{L}} d^*_{\mathrm{L}}}}$. However, this is no longer true if we renormalize $m^2_{\widetilde{d_{\mathrm{L}} d^*_{\mathrm{L}}}}$: its value stays (4) at the high scale, but its evolution down to the M_W scale cannot be diagonal due to the $h_u Q H_1 u^c$ term in the superpotential (1). Hence the 3×3 mass matrix of $m^2_{\widetilde{dd^*}}$ renormalized to the M_W scale reads

$$m^2_{\widetilde{d_{\mathrm{L}} d^*_{\mathrm{L}}}} (q^2 = M_w^2) = m_d m_{d^+} + m^2 \times 1 + c m_u m_u^+, \quad (6)$$

where the coefficient c can be computed by solving the set of renormalization group equations for the evolution of the SUSY quantities. From (6) we see that the simultaneous diagonalization of $m^2_{\widetilde{d_{\mathrm{L}} d^*_{\mathrm{L}}}}$ and $m_d m_{d^+}$ is no longer possible due to the presence of the $c m_u m_u^+$ term. The flavor change is proportional to c and to the usual (CKM) angles. In a basis where d_{L}–$\widetilde{d^+_{\mathrm{L}}}$– $\widetilde{g}$ couplings are flavor diagonal, the flavor mixing occurs in the squark propagators. The above remark can be summarized in the following schematic way:

$$\tilde{d}_{iL} \overset{\times}{\underset{\Delta^{ij}_{LL}}{--}} \tilde{d}_{jL} \quad \to \quad \Delta^{ij}_{\mathrm{LL}} = c(V.[m_u^{\mathrm{diag}}]^2.V^\dagger)_{ij}. \quad (7)$$

For the $\mathrm{L} \to \mathrm{R}$ transitions, we have

$$\tilde{d}_{iL} \overset{\times}{\underset{\Delta^{ij}_{LR}}{--}} \tilde{d}^c_{jL} \quad \to \quad \Delta^{ij}_{\mathrm{LR}} = \Delta^{bs}_{\mathrm{LR}}. \quad (8)$$

The quantities Δ_{ij} are mass insertions connecting flavors i and j along a sfermion propagator and the indices L,R refer to the helicity of the fermion partners. There are three types of sfermions mixing: Δ_{LL}, Δ_{RR} and Δ_{LR}. In the MSSM case with universal soft SUSY breaking (MSUGRA), there exists a kind of hierarchy among mass insertions; that is, $(\Delta_{\mathrm{LL}})_{ij} \gg (\Delta_{\mathrm{LR}})_{ij} \gg (\Delta_{\mathrm{RR}})_{ij}$. This is no longer true if flavor changing is produced by another kind of "initial" conditions. Then, generally, nothing can be said about the hierarchy of these three contributions. In that case, one needs a model-independent parameterization of the flavor changing (FC) and CP quantities in SUSY to test variants of the universal MSSM. The chosen parameterization is the mass insertion approximation [6–8]. It concerns the most peculiar source of FCNC SUSY contributions that do not arise from the mere supersymmetrization of the FCNC in the SM. They originate from the FC coupling of gluinos, neutralinos and charginos to fermions and sfermions. One chooses a basis for fermions and sfermions states where all couplings of these particles to gauginos are flavor diagonal, while the FC originates from non-diagonal sfermion mass terms in propagators. Denoting by Δ the off-diagonal terms in the sfermions mass matrix (i.e. the mass terms relating sfermions of the same electric charge, but different flavor), the sfermion propagators can be expanded as a series in $\delta = \Delta/\widetilde{m}^2$ where $\widetilde{m}$ is an average sfermions mass and a typical scale of the SUSY breaking. As long as the ratio of non-diagonal entries (Δ) to the average squark mass is a small parameter [5], the first term in the expansion, obtained from the non-diagonal insertion of mass between two diagonal squark propagators, represents a reasonable approximation. This method has the advantage that one does not need the full diagonalization of the sfermion mass matrices. So, from the FCNC experimental data we may derive upper bounds on the different δ's.

In the following section, we give the explicit expression of the amplitudes associated to the gluinos, neutralinos and charginos contributions to the decay $b \to s\gamma$ [2,10, 11].

3 Amplitudes contributing to the decay $b \to s\gamma$

The MSSM Feynman rules used for the calculation of the amplitudes can be found in [9]. The calculation of these amplitudes is done with these Feynman rules and the mass insertion approximation. Supersymmetric penguin diagrams contributing to the decay $b \to s\gamma$ are

(1) gluinos (pengluinos): Fig. 1a,b;
(2) neutralinos (penneutralinos): Fig. 2a,b;
(3) charginos (pencharginos): Fig. 3a,b; Fig. 4a,b; Fig. 5a,b.

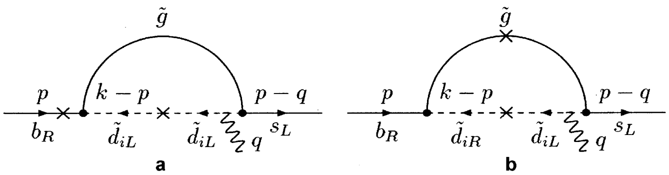

Fig. 1a,b. Gluino contribution (pengluinos)

These diagrams induce the effective operator $O_{\rm LR} = m_b \varepsilon_\mu(q)\overline{s(p-q)}\sigma^{\mu\nu}q_\nu P_{\rm R} b(p)$; q is the outgoing momentum of the photon.

Now we can give the explicit expressions of the amplitudes associated with the supersymmetric penguin diagrams.

3.1 The pengluinos

From the diagram illustrated in Fig. 1a, we obtain

$$T'_{\widetilde{g_{\rm LL}}} = e_D C_2(R)\alpha_{\rm s}\frac{1}{\sqrt{\pi}}\sqrt{\alpha}\frac{\delta_{\rm LL}}{M_D^2} \qquad (9)$$
$$\times\ \varepsilon_\mu(q)\overline{s(p-q)}\sigma^{\mu\nu}q_\nu P_{\rm R}[pH(X_{\widetilde{g}}) + qH(X_{\widetilde{g}})]b(p),$$

where D_i with $i = 1, \ldots, 6$ are squark down mass eigenstates, e_D is the electric charge of the squark D; $X_{\widetilde{g}} = M_{\widetilde{g}}^2/M_D^2$ and the function $H(X_{\widetilde{g}})$ is given in the appendix; $\delta_{\rm LL}$ is the mass insertion connecting flavors b and s with the helicity L:

$$\delta_{\rm LL} = \sum_{i=1}^{6}\frac{(M_{Di}^2 - M_D^2)Z_D^{si*}Z_D^{bi}}{M_D^2} = \frac{\Delta_{\rm LL}}{M_D^2}, \qquad (10)$$

in which Z_D is a mixing matrix defined by

$$\mathrm{diag}(M_{D1}^2, \ldots, M_{D6}^2) = Z_D^+\begin{pmatrix} M_{\rm LL}^2 & M_{\rm LR}^{2+} \\ M_{\rm LR}^2 & M_{\rm RR}^2 \end{pmatrix} Z_D;$$

M_D^2 is the average squark down mass and $C_2(R) = 4/3$ (fundamental representation). So the $T'_{\widetilde{g_{\rm LL}}}$ expression becomes

$$T'_{\widetilde{g_{\rm LL}}} = e_D C_2(R)\alpha_{\rm s}\frac{\delta_{\rm LL}}{M_D^2\sqrt{\pi}}\sqrt{\alpha}\varepsilon_\mu(q)\overline{s(p-q)}$$
$$\times\ \sigma^{\mu\nu}q_\nu P_{\rm R}[pH(X_{\widetilde{g}}) + qH(X_{\widetilde{g}})]b(p).$$

From the diagram drawn in Fig. 1b we have

$$T'_{\widetilde{g_{\rm LR}}} = e_D C_2(R)\alpha_{\rm s}\frac{M_{\widetilde{g}}\delta_{\rm LR}}{M_D^2\sqrt{\pi}}\sqrt{\alpha}M_1(X_{\widetilde{g}})\varepsilon_\mu(q)\overline{s(p-q)}$$
$$\times\ \sigma^{\mu\nu}q_\nu P_{\rm R} b(p),$$

with

$$\delta_{\rm LR} = \sum_{i=1}^{6}\frac{(M_{Di}^2 - M_D^2)Z_D^{si*}Z_D^{(b+3)i}}{M_D^2} = \frac{\Delta_{\rm LR}}{M_D^2}.$$

Thanks to the experimental limits, it will be possible to put upper bounds on the different δ's; that is, on the non-diagonal terms in the sfermion mass matrix.

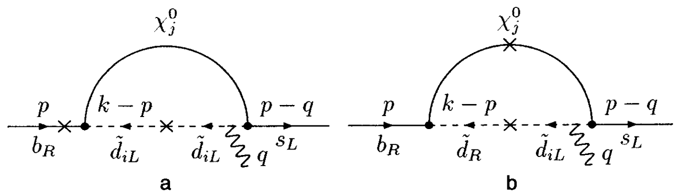

Fig. 2a,b. Neutralino contributions (penneutralinos)

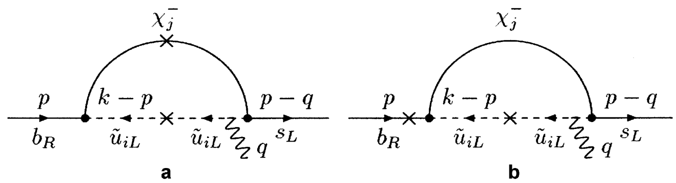

Fig. 3a,b. Chargino contributions with photon coupling to up squark and mass insertion LL

3.2 The penneutralinos

The penneutralinos are illustrated in Fig. 2a,b.

(1) For diagram (a)

$$T'_{\chi^0_{{\rm LL}j}} = e_D\alpha_w\frac{\delta_{\rm LL}}{2\cos^2(\theta_{\rm W})M_D^2\sqrt{\pi}}\sqrt{\alpha}(z_{{\rm L}\chi_j^0})\overline{s(p-q)}$$
$$\times\ \varepsilon_\mu(q)\sigma^{\mu\nu}q_\nu P_{\rm R}[pH(X_{0j}) - qH(X_{0j})/3]b(p),$$

$j = 1, \ldots, 4$ the four neutralinos indices, $X_{0j} = M_{\chi_j^0}^2/M_D^2$, with $M_{\chi_j^0}$ the neutralino mass, and

$$z_{{\rm L}\chi_j^0} = \left|\frac{1}{3}Z_N^{1j}\sin\theta_{\rm W} - Z_N^{2j}\cos\theta_{\rm W}\right|^2.$$

Clearly, $z_{{\rm L}\chi_j^0}$ is less than or equal to 1. In MSUGRA, for example, we will have $z_{{\rm L}\chi_1^0} \approx \sin^2(\theta_{\rm W})/9$, because $Z_N^{11} \approx 1$ for the lightest neutralino (bino-like), and $z_{{\rm L}\chi_2^0} \cong 0.8$ for $Z_N^{22} \approx 1$.

(2) For diagram (b)

$$T'_{\chi^0_{{\rm RL}j}} = -e_D\alpha_w\frac{\sin(\theta_{\rm W})\delta_{\rm LR}M_{\chi_j^0}}{3\cos^2(\theta_{\rm W})M_D^2\sqrt{\pi}}$$
$$\times\ \sqrt{\alpha}(z_{Rx_j^0})M_1(X_{0j})\overline{s(p-q)}\varepsilon_\mu(q)\sigma^{\mu\nu}q_\nu P_{\rm R}b(p),$$

with

$$z_{{\rm R}\chi_j^0} = \left(\frac{1}{3}Z_N^{1j*}\sin\theta_{\rm W} - Z_N^{2j*}\cos\theta_{\rm W}\right)Z_N^{1j*},$$

where, in MSUGRA, we can have $z_{{\rm R}\chi_2^0} \approx 1$.

3.3 The pencharginos

Due to the photon–chargino coupling, there are six diagrams. For the mass insertion LL, the four diagrams illustrated in Fig. 3a,b and Fig. 4a,b contribute. When the

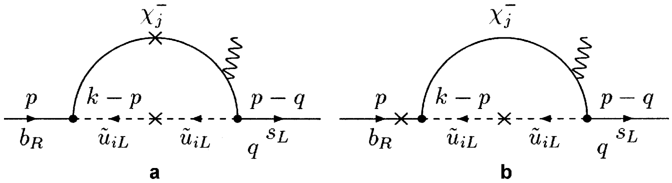

Fig. 4a,b. Chargino contributions with photon coupling to the chargino and mass insertion LL

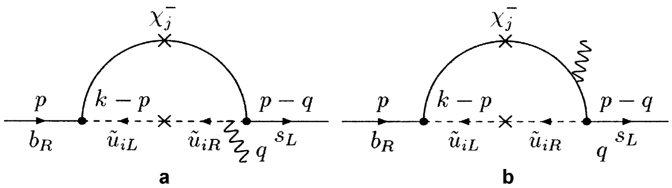

Fig. 5a,b. Chargino contributions with mass insertion LR

helicity flip is realized in the quark b external line, only the wino component of the chargino is concerned in the calculation of the amplitude (diagrams b). But in the case where the helicity flip is realized on the chargino line the higgsino components are taken into account (diagrams a). From the penchargino diagrams, Fig. 3a,b, where the photon is coupled to the squarks, we obtain

(1) for diagram (a)

$$T'_{\chi^-_{\mathrm{LL}j}} = e_u \alpha_w \frac{m_b \delta_{\mathrm{LL}\chi_j} M_{\chi_j^-} Z_{1j}^{+*} Z_{2j}^{-*}}{M_w \cos(\beta) M_U^2 \sqrt{2\pi}} \times \sqrt{\alpha} M_1(X_j) \overline{s(p-q)} \varepsilon_\mu(q) \sigma^{\mu\nu} q_\nu P_{\mathrm{R}} b(p),$$

where $j = 1, 2$ are the two chargino states, $X_j = M^2_{\chi_j^-}/M_U^2$ and $M_{\chi_j^-}$ is the j chargino mass, $e_u = 2/3$ is the squarks up electric charge, M_U the squark up average mass. Also we have

$$\delta_{\mathrm{LL}\chi_j} = \sum_{i=1}^{6} \sum_{J=1}^{3} \sum_{K=1}^{3} (1 - \delta_{JK}) \frac{(M_{Ui}^2 - M_U^2)}{M_U^2} \times Z_U^{Ki} V_{sK} Z_U^{Ji*} V_{bJ}^*, \quad (11)$$

in which J and K run over the three generations of squarks and U_i with $i = 1, \ldots, 6$ are up squarks mass eigenstates. As in δ_{LL}, $\delta_{\mathrm{LL}\chi j}$ contains squark mixing factors Z, but in addition, there are some known Cabbibo quarks mixing factors (e.g. V_{bc}).

(2) For diagram (b)

$$T'_{\chi^-_{\mathrm{LL}j}} = e_u \alpha_w \frac{\delta_{\mathrm{LL}\chi_j} Z_{1j}^{+*} Z_{1j}^{+}}{M_U^2 \sqrt{\pi}} \sqrt{\alpha} \overline{s(p-q)} \varepsilon_\mu(q) \times \sigma^{\mu\nu} q_\nu P_{\mathrm{R}} [pH(X_j) + qH(X_j)/3] b(p).$$

In MSUGRA for the lightest chargino, we have $Z_{1j}^{+*} \approx 1$ when $Z_{2j}^{-*} \approx 0$; in such a case, we remark that only diagram (b) (wino component of the chargino) contributes to the amplitude.

The penchargino illustrated in Fig. 4a,b, where the photon is coupled to the chargino, give

(1) for diagram (a)

$$T'_{\chi^-_{\mathrm{LL}j}} = -\alpha_w \frac{m_b \delta_{\mathrm{LL}\chi_j} M_{\chi_j^-} Z_{1j}^{+*} Z_{2j}^{-*}}{M_w^2 \cos(\beta) M_U^2 \sqrt{2\pi}} \times \sqrt{\alpha} F(X_j) \overline{s(p-q)} \varepsilon_\mu(q) \sigma^{\mu\nu} q_\nu P_{\mathrm{R}} b(p);$$

(2) for diagram (b)

$$T'_{\chi^-_{\mathrm{LL}j}} = \alpha_w \frac{\delta_{\mathrm{LL}\chi_j} Z_{1j}^{+*} Z_{1j}^{+}}{M_U^2 \sqrt{\pi}} \sqrt{\alpha} \overline{s(p-q)} \varepsilon_\mu(q) \times \sigma^{\mu\nu} q_\nu P_{\mathrm{R}} [pG(X_j) + qG(X_j)/2] b(p).$$

We define the fraction of gaugino in the chargino j by $z_{\chi \mathrm{L}j} = Z_{1j}^{+*} Z_{1j}^{+}$. The $F(X_j)$ and $G(X_j)$ are given in Appendix A. As above, in MSUGRA, only diagram (b) will contribute to the amplitude.

The LR mass insertion for the chargino contribution is illustrated in Fig. 5a,b. Only the higgsino components of the chargino contributes to the amplitude. Therefore, due to the s_{L}–χ_j^-–$U_{i\mathrm{R}}$ coupling giving a factor $U^J Z_U^{(J+3)i} Z_{2j}^{+*} P_{\mathrm{R}} V_{sJ}$, where U^J is a Yukawa coupling proportional to the associated quark mass, the top quark contribution overwhelms the up and charm ones.

From diagram (a) in Fig. 5 we thus obtain

$$T'_{\chi^-_{\mathrm{LR}j}} = -e_u \alpha_w \frac{m_t m_b \delta_{\mathrm{LR}\chi_j} z_{\mathrm{R}j} M_{\chi_j^-}}{M_W^2 \cos(\beta) \sin(\beta) M_U^2 \sqrt{\pi}} \times \sqrt{\alpha} M_1(X_j) \overline{s(p-q)} \varepsilon_\mu (q \sigma^{\mu\nu} q_\nu P_{\mathrm{R}} b(p),$$

where[1]

$$\delta_{\mathrm{LR}\chi_j} = \sum_{i=1}^{6} \frac{(M_{Ui}^2 - M_U^2)}{M_U^2} Z_U^{(t+3)i} V_{st} Z_U^{ti*} V_{bt}^*, \quad (12)$$

and for diagram (b)

$$T'_{\chi^-_{\mathrm{LR}j}} = \alpha_w \frac{m_t m_b \delta_{\mathrm{LR}\chi_j} z_{\mathrm{R}j} M_{\chi_j^-}}{2 M_W^2 \cos(\beta) \sin(\beta) M_U^2 \sqrt{\pi}} \times \sqrt{\alpha} F(X_j) \overline{s(p-q)} \varepsilon_\mu(q) \sigma^{\mu\nu} q_\nu P_{\mathrm{R}} b(p),$$

with $z_{\mathrm{R}j} = Z_{2j}^{+*} Z_{2j}^{-*}$, the fraction of higgsino in the chargino j, its greatest value is 1 and the minimum value 1/2 for one of the two charginos. In the following section we give the explicit expression of the branching ratio for the decay $b \to s\gamma$ and the upper bounds on the mass insertions.

[1] It would be desirable to pull out CKM factors from the δ's, so that they only reflect squark properties. While this is arguably possible for $\delta_{\mathrm{LR}\chi}$, it requires non-trivial assumptions for $\delta_{\mathrm{LL}\chi}$, which is why we kept the CKM factors in the definition of both

Table 1.

α_s	α	α_w	$\sin^2(\theta_W)$	$BR(b \to s\gamma)$	M_W	m_b	m_t	τ_B
0.12	1/127.9	α/s_W^2	0.2315	$1 \to 4 \times 10^{-4}$	80.41	4.5	170	1.49×10^{-12} s

Table 2. Limits on the off-diagonal terms δ_{LL} and δ_{LR} for down squarks with $M_{\tilde{q}} = 300$ GeV (or $M_{\tilde{q}} = 500$ GeV), coming from gluino and neutralino contributions

$M_{\tilde{g}}$	$X_{\tilde{g}}$	δ_{LL}	δ_{LR}	$M_{\chi_1^0}$	X_0	$\delta_{LL} z_{L\chi_1^0}$	$\delta_{LR} z_{R\chi_1^0}$
300	1	2.96	10^{-2}	50	3×10^{-2}	7.1	0.34
	(0.36)	(8.2)	(2.7×10^{-2})		(10^{-2})	(19.7)	(0.94)
600	4	9.5	1.8×10^{-2}	100	10^{-2}	8.4	0.25
	(1.44)	(26.4)	(4.9×10^{-2})		(4×10^{-2})	(23.25)	(0.7)
800	7	17.6	2.6×10^{-2}	130	0.19	9.8	0.23
	(2.56)	(48.8)	(7.2×10^{-2})		$(7 \times 10^{-2)}$	(27.3)	(0.64)

Table 3. Limits on the off-diagonal terms δ_{LL} and δ_{LR} for up squarks with $M_{\tilde{q}} = 300$ GeV (or $M_{\tilde{q}} = 500$ GeV), coming from charginos contributions

M_{χ^-}	X	$\delta_{LL\chi} z_{\chi L1}$	$\delta_{LR\chi} z_{R1}$	$\delta_{LR\chi} z_{R1}$	$\delta_{LR\chi} z_{R1}$	$\delta_{LR\chi} z_{R1}$	$\delta_{LR\chi} z_{R1}$
	$\tan(\beta) = 2$	$M_{U=M_q}$		$\tan(\beta) = 5$	$\tan(\beta) = 10$	$\tan(\beta) = 20$	$\tan(\beta) = 40$
100	0.1	0.57	0.08	0.04	0.02	0.01	0.0051
	(4×10^{-2})	(1.58)	(0.09)	(0.045)	(0.023)	(0.011)	(0.0058)
200	0.44	1.23	1.6	0.76	0.39	0.2	0.099
	(0.16)	(3.41)	(0.2)	(0.095)	(0.049)	(0.025)	(0.012)
300	1	2.34	0.3	0.14	0.075	0.03	0.019
	(0.36)	(6.5)	(0.83)	(0.4)	(0.2)	(0.1)	(0.052)

4 The expression for $BR(b \to s\gamma)$ and the upper bounds on the mass insertions δ

The decay $b \to s\gamma$ is very interesting because the rare B decays represent a good test for new physics beyond SM since they are not affected appreciably by uncertainties due to long distance effects.

The branching ratio [10] is

$$BR(b \to s\gamma) = \frac{BR(B \to X_s\gamma)}{BR(B \to X_c e\overline{\nu_e})} = \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\overline{\nu_e})},$$

where $b \to ce\overline{\nu_e}$ is dominant; then $BR(b \to s\gamma) = \Gamma(b \to s\gamma)\tau_B$. The explicit expression of $BR(b \to s\gamma)$ obtained from the calculation exposed in Sect. 3 is

$$\begin{aligned} BR(b \to s\gamma) = \frac{m_b^3 \alpha \tau_B}{16\pi^2} \Bigg| & \frac{m_b \alpha_s e_D C_2(R)}{M_D^2} \delta_{LL} H(X_{\tilde{g}}) \\ & + \frac{\alpha_s e_D C_2(R) M_{\tilde{g}}}{M_D^2} \delta_{LR} M_1(X_{\tilde{g}}) \\ & - \frac{e_D \alpha_w M_{\chi_j^0} \sin^2(\theta_W) z_{R\chi_j^0}}{9M_D^2} \delta_{LR} M_1(X_{0j}) \\ & + \frac{e_D \alpha_w m_b \sin^2(\theta_W) z_{L\chi_j^0}}{18 M_D^2 \cos^2(\theta_W)} \delta_{LL} H(X_{0j}) \\ & + \frac{m_b \alpha_w}{M_U^2} (G(X_j) + e_U H(X_j)) \delta_{LL\chi_j} \\ & + \frac{\alpha_w m_b m_t M_{\chi_j^-}}{M_w^2 \cos(\beta)\sin(\beta) M_U} \\ & \times \left(\frac{F(X_j)}{2} - e_U M_1(X_j) \right) \delta_{LR\chi_j^-} \Bigg|^2 \end{aligned} \quad (13)$$

By imposing the condition that each individual term, in the above equation, does not exceed in absolute value the experimental data of $BR(b \to s\gamma)$; that is, 1–4 $\times 10^{-4}$ (which includes the QCD uncertainties following [4]), we give upper bounds on the different δ.

We have chosen the values for the supersymmetric particles from the experimental data given in [12]. Moreover, we have imposed the following conditions.

(1) The average squark mass: $M_U = M_D = M_{\tilde{q}}$.
(2) For the neutralino masses: we choose the LSP mass, with $M_{\chi_1^0} \approx M_{\tilde{g}}/6$ (GUT relation n).
(3) The lightest stop mass: $M_{\tilde{t}} \geq M_{\chi_1^0} + 30$ GeV.
(4) The chargino mass: $M_{\chi_1^-} \approx 2M_{\chi_1^0}$ (GUT).

Otherwise, we have $X_{\tilde{g}} = M_{\tilde{g}}^2/M_{\tilde{q}}^2$, $X_0 = M_{\chi^0}^2/M_{\tilde{q}}^2$, $X = M_{\chi^-}^2/M_{\tilde{q}}^2$. The others experimental data chosen are defined in Table 1

The results are given, in Tables 2 and 3, for different $\tan(\beta)$ values (i.e. $\tan(\beta) = 2$, 5, 10, 20 and 40) and for two values of $M_{\tilde{q}}$: $M_{\tilde{q}} = 300$ GeV and $M_{\tilde{q}} = 500$ GeV

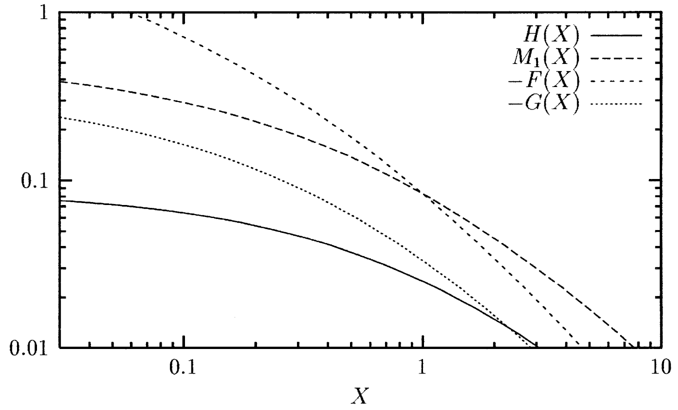

Fig. 6. The different functions encountered in the evaluation of the amplitudes, for the available values of $X = M^2_{\rm gaugino}/M^2_{\rm squarks}$ on a logarithmic scale

(the corresponding results are denoted in parentheses in the tables). We have obtained the LL insertion limits by imposing $\mathrm{BR}(b \to s\gamma) = 4\times10^{-4}$ and for the LR insertion $\mathrm{BR}(b \to s\gamma) = 2 \times 10^{-4}$ to be consistent with [6] in the gluino case. For the chargino contribution we take $M_U = M_{\tilde q}$.

From the results obtained we remark the following.

(1) Because of our use of the mass insertions, we are limited to $\delta < 1$ for the branching ratio expression (13) to make sense. When larger than one, the experimental bounds on δ like $\delta_{\rm LL} < 2.96$ thus really mean that the maximal effect of such terms (for $\delta_{\rm LL} \sim 1$) only contributes a fraction of about 1/9th of the experimental bound.
(2) In the gluino case, $\delta_{\rm LL}$ is more sensitive to the gluino mass than $\delta_{\rm LR}$, because this last contribution has an amplitude enhancement factor of $M_{\tilde g}$. Then the dependence on $M_{\tilde g}$ of the $H(X_g)$ function is partially compensated (see the definitions in Appendix A and the plot of Fig. 6).
(3) For the neutralino, the limit on $\delta_{\rm LL}$ is less sensitive to the neutralino mass, thanks to the small values of X_0 contained in the same $H(X_0)$ function. However, even for the upper value for $z_{{\rm L}\chi^0_j} = 1$ (e.g. for $j = 2$), the limit cannot be more competitive than the gluino limit, except if $M_{\chi^0}/M_{\tilde g}$ is smaller than in the MSUGRA models. The $\delta_{\rm LR}$ limits decrease weakly with the enhancement of M_{χ^0}, due to the additional power of M_{χ^0} in the amplitude and the fact that the function $M_1(X_0)$ is fairly constant for small X_0 (see Appendix A and the plot of Fig. 6).
(4) The chargino contribution is the only one constraining the differences between the up squark masses. However the expressions for $\delta_{{\rm LL}\chi}$ and $\delta_{{\rm LR}\chi}$ contain Cabbibo mixing factors. For $\delta_{{\rm LR}\chi}$, the dominating top contribution allows one to extract a simple factor of $V_{ts}V_{tb} \approx 1/30$. The first remark above then applies for interpreting our mass insertions results, once $\delta_{{\rm LL}\chi} > 0.03$. Nevertheless, we obtain a limit that quickly becomes more constraining with increasing $\tan(\beta)$. For $\delta_{{\rm LL}\chi}$, the factorization of CKM elements cannot be so easily justified: if for instance there is a large mixing between the squarks 2 and 3, the leading CKM factors are of order 1, and the expression (11) for $\delta_{{\rm LL}\chi}$ with up squarks becomes the same as $\delta_{\rm LL}$ for down squarks in (10). The sensitivity on $M_{\tilde q}$ of the bounds is about the same factor of 3 for the chargino $\delta_{{\rm LL}\chi}$ as for the neutralino $\delta_{\rm LL}$ and $\delta_{\rm LR}$, while the chargino $\delta_{{\rm LR}\chi}$ is less sensitive for small chargino masses.
(5) The limits in Tables 2 and 3 can easily be generalized for different values of squark and gaugino masses: they are inversely proportional to the functions given in Appendix A and plotted in Fig. 6.
(6) Finally, if following [13] we use the latest CLEO data [14] and subtract the SM contribution [15], we obtain a tighter bound, in the absence of cancellations between the various contributions that was assumed throughout this paper. The various results in the tables are then reduced by a factor of about 20.

5 Conclusion

We have given explicit expressions for the amplitudes associated to the supersymmetric contributions to the decay $b \to s\gamma$ in the context of supersymmetric extensions of SM with non-universal soft SUSY breaking terms. The model-independent parameterization which we have chosen is the mass insertion approximation. From the FCNC experimental data, we have derived upper bounds on the different δ's. The contribution from the chargino and neutralino exchanges are less sensitive to the gaugino mass than the gluino contribution.

Acknowledgements. This work was supported in part by the GdR Supersymetrie of the French CNRS.

A Appendix

$$H(X) = \frac{-1 + 9X + 9X^2 - 17X^3 + 6X^2(X\ln X + 3\ln X)}{12(X-1)^5},$$

$$M_1(X) = \frac{1 + 4X - 5X^2 + 2X(X+2)\ln(X)}{8(X-1)^4} = L(X)/2,$$

$$F(X) = \frac{5 - 4X - X^2 + 2\ln(X) + 4X\ln(X)}{2(X-1)^4},$$

$$G(X) = \frac{1 + 9X - 9X^2 - X^3 + 6X(1+X)\ln(X)}{3(X-1)^5}.$$

References

1. S. Bertolini et al., Phys. Lett. B **192**, 437 (1987)
2. M. Gronau et al., Phys. Rev. D **55**, 2845 (1997)
3. F. Gabbiani et al., Nucl. Phys. B **322**, 235 (1989)
4. R. Grigjanis et al., Phys. Rep. **228**, 3 (1998)
5. E. Gabrielli et al., Phys. Lett. B **374**, 80 (1996)
6. F. Gabbiani et al., Nucl. Phys. B **477**, 321 (1996)
7. S. Bertolini et al., Nucl. Phys. B **353**, 591 (1991)
8. J.S. Hagelin et al., Nucl. Phys. B **415**, 293 (1994)

9. J. Rosiek, Phys. Rev. D 41 **11**, 3464 (1990)
10. N. Oshimo, Nucl. Phys. B **404**, 20 (1993)
11. S. Bertolini et al., Z. Phys. C **67**, 513 (1995)
12. ALEPH collaboration, 99-078, Conf.99-050; ALEPH collaboration, 99-072, Conf.98-041; A. Djouadi et al., Phys. Rev. D **61**, 095006 (2000); ALEPH collaboration, Phys. Lett. B **413**, 431 (1997)
13. P. Gambino et al., JHEP **0009**, 001 (2000)
14. CLEO collaboration, CLEO CONF 98-17; ALEPH collaboration, R. Barate et al., Phys. Lett. B **429**, 169 (1998)
15. K. Chetyrkin et al., Phys. Lett. B **400**, 206 (1997); erratum Phys. Lett. B **425**, 414 (1998); A.J. Buras et al., Phys. Lett. B **414**, 157 (1997); erratum Phys. Lett. B **434**, 459 (1998); A.L. Kagan et al., Eur. Phys. J. C **7**, 5 (1999)
16. T. Besmer, C. Greub, T. Hurth, Nucl. Phys. B **609**, 359 (2001) and hep-ph/0111389

Note added in proof: While this work was being refereed, a general study of $B \to X_s\gamma$ appeared [16] with interesting results on the interferences between various contributions, including the ones presented here. For the parameters studied there ($\mu = 300\,\mathrm{GeV}$, $M_{\tilde{q}} = 500\,\mathrm{GeV}$, $M_2 = 100 \to 230\,\mathrm{GeV}$), we agree that there is no constraint on up squarks from Table 3: the light chargino is a gaugino with $z_{\mathrm{R}1} \ll 1$ and the heavy chargino gives the last line in parentheses, with $z_{\mathrm{R}2} = 1$. Taking the Cabbibo factors into account, the strongest bound on the off-diagonal element $\delta_{u,\mathrm{LR},23}$ is just about 1 for $\tan\beta = 50$, which means no constraint in the logic of mass insertion.